Radial migration of the Sun in galactic disk

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ABSTRACT

Physics of the gravitational effect of the galactic bar and spiral structure is presented. Physical equations differ from the conventionally used equations.

Application to the motion of the Sun is treated. The speed of the Sun is taken to be consistent with the Oort constants.

Galactic radial migration of the Sun is less than ± 0.4 *kpc* for the four-armed spiral structure. The Sun remains about 75 % of its existence within galactocentric distances (7.8 − 8.2) *kpc* and the results are practically independent on the spiral structure strength. Thus, the radial distance changes only within 5% from the value of 8 *kpc*.

Galactic radial migration of the Sun is less than ± (0.3 − 1.2) *kpc*, for the two-armed spiral structure. The Sun remains (29 − 95)% of its existence within galactocentric distances (7.8 − 8.2) *kpc* and the results strongly depend on the spiral structure strength and the angular speed of the spiral arms. The radial distance changes within (3.8 − 15.0)% from the value of 8 *kpc*.

If observational arguments prefer relevant radial migration of the Sun, then the Milky Way is characterized by the two-arm spiral structure.

Key words: Galaxy – galactic bar – galactic spirals – motion of the Sun – equation of motion.

1 INTRODUCTION

Orbital evolution of the Sun in our galaxy, Milky Way, is important for understanding of galactic evolution, its dynamics, kinematics and chemical composition. It is also relevant for understanding of evolution of the Solar System, e.g. the effect of the galactic tides on the evolution of the Öpik - Oort cloud of comets. Several papers on the gravitational effect of the galactic bar on motion of the Sun in the Galaxy appeared recently, e.g. Minchev & Famaey (2010), Famaey & Minchev (2010), Minchev et al. (2010), Minchev et al. (2011). One of the important results of the papers shows that the Sun can radially migrate in the disk of the Galaxy.

Minchev & Famaey (2010), Famaey & Minchev (2010) present the change of the solar galactocentric distance. The distance may change within several kiloparsecs.

If we are interested in real galactic radial migration of the Sun, we have to treat physical equation of motion. The first aim of this paper is to find the equation of motion, to put the relevant equations on a firm physical basis. The second aim of this paper is to use the physical equations in performing calculations similar to Minchev & Famaey (2010). We are interested in radial motion (radial migration) of the Sun in the Galaxy.

Section 2 presents multipole expansion of the galactic

potential and the concentration is paid to the quadrupole term, important for the galactic bar. The relevant quantities found in the monopole and quadrupole terms are discussed in Section 3, where the values of the quantities are calculated for several models of the galactic bar. Section 4 offers the equation of motion for an object under the gravitational influence of the galactic disk, halo and the monopole and quadrupole terms of the galactic bar (the monopole term corresponds to the conventional idea of the galactic bulge). Section 5 presents numerical results for the radial migration of the Sun. Finally, discussion and conclusion can be found in Sections 6 and 7.

2 MULTIPOLE EXPANSION

Having a mass density distribution $\varrho(\vec{r})$, the gravitational potential at a point $\vec{r}$ is

$$\Phi(\vec{r}) = - G \int \frac{\varrho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} \; d^3\vec{r}\,' \; , \qquad (1)$$

where G is the gravitational constant. Making an expansion in terms of the ratio $|\vec{r}\,'|/|\vec{r}| \equiv r'/r$, we can write

$$\Phi(\vec{r}) = \sum_{j=0}^{\infty} \Phi^{(j)}(\vec{r}) , \tag{2}$$

and,

$$\Phi^{(0)}(\vec{r}) = - G \frac{1}{r} \int \varrho(\vec{r}\,') \, d^3\vec{r}\,' = - \frac{G \, M}{r} , \tag{3}$$

where M is the total mass concentrated in a small region so that the condition $|\vec{r}\,'|/|\vec{r}| \ll 1$ is fulfilled. Other terms of the expansion are $\Phi^{(1)}(\vec{r}) = 0$, and, the quadrupole term is (Einstein's summation convention is used)

$$\begin{aligned}
\Phi^{(2)}(\vec{r}) &= - \frac{G}{2 \, r^3} \, Q_{ij} \, n_i \, n_j , \\
Q_{ij} &= \int [3(\vec{r}\,')_i \, (\vec{r}\,')_j - |\vec{r}\,'|^2 \delta_{ij}] \, \varrho(\vec{r}\,') \, d^3\vec{r}\,' , \\
n_i &= (\vec{r})_i / r
\end{aligned} \tag{4}$$

and δ_{ij} is the Kronecker delta. Since $\Phi^{(2)}(\vec{r})$ is given by a contraction of two tensors of the second order, Q_{ij} and $n_i \, n_j$, it is an invariant independent on the coordinate basis. The numerical calculations can be done in a special coordinate basis, the primed coordinates, e.g., defined by the relations describing rotation

$$\begin{aligned}
x' &= + x \, \cos\alpha + y \, \sin\alpha , \\
y' &= - x \, \sin\alpha + y \, \cos\alpha , \\
z' &= z .
\end{aligned} \tag{5}$$

In the case of a galactic bar we will use

$$\alpha = \Omega_b \, t + \alpha_0 , \tag{6}$$

where $|\Omega_b|$ is the magnitude of the angular velocity of the bar's rotation. In reality, Ω_b is negative and the negative sign denotes the observational fact that rotation is negative, i.e., clockwise.

Eqs. (4)-(6) yield

$$\begin{aligned}
\Phi^{(2)}(x,y,z) &= - \frac{G}{2 \, r^3} \times X_\Phi , \\
X_\Phi &= Q_{11} n_1^2 + Q_{22} n_2^2 - (Q_{11} + Q_{22}) \, n_3^2 ,
\end{aligned} \tag{7}$$

where primes above the Q_{11} and Q_{22} terms are omitted, for the purpose of brevity. The coordinate system is chosen in the way that $Q_{ij} = 0$ if $i \neq j$ and some symmetry exists. The mass density is an even function of coordinate arguments,

$$\begin{aligned}
Q_{11} &= \int_{-\infty}^{\infty} \rho(x,y,z) \left(2x^2 - y^2 - z^2\right) \, dxdydz , \\
Q_{22} &= \int_{-\infty}^{\infty} \rho(x,y,z) \left(2y^2 - x^2 - z^2\right) \, dxdydz
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
n_1^2 &= \left(x^2 \cos^2\alpha + y^2 \sin^2\alpha + 2xy \sin\alpha \cos\alpha\right)/r^2 , \\
n_2^2 &= \left(x^2 \sin^2\alpha + y^2 \cos^2\alpha - 2xy \sin\alpha \cos\alpha\right)/r^2 , \\
n_3^2 &= z^2/r^2 .
\end{aligned} \tag{9}$$

Inserting Eqs. (9) into Eq. (7) one obtains

$$\begin{aligned}
\Phi^{(2)}(x,y,z) &= - \frac{G \, Q_{11} \left(1 - Q_{22}/Q_{11}\right)}{4 \, r^5} \times X_{\Phi 2} \\
X_{\Phi 2} &= \frac{1 + Q_{22}/Q_{11}}{1 - Q_{22}/Q_{11}} \left(x^2 + y^2 - 2 \, z^2\right) \\
&\quad + \left(x^2 - y^2\right) \cos\left(2\alpha\right) + 2 \, x \, y \, \sin\left(2\alpha\right) , \\
\alpha &= \Omega_b \, t + \alpha_0 ,
\end{aligned} \tag{10}$$

if also Eq. (6) is added. As for the Milky Way, the value of α_0 is about -25 degrees (compare with Majaess 2010; the real value may differ in more than 10 degrees) and the value of the angular velocity is $\Omega_b = -55.5 \; km \; s^{-1} \; kpc^{-1}$ (compare with Minchev & Famaey 2010), t is the time.

2.1   Discussion on Eq. (10)

Our result represented by Eq. (10) differs from the result conventionally used by other authors. The conventional result is (see, e.g., Minchev & Famaey 2010, Dehnen 2000)

$$\begin{aligned}
\Phi^{(2)}_{conv} &= \frac{1}{2} Q_T \, v_c^2 \, \cos\left[2(\phi - \Omega_b t)\right] \times X_{\Phi 2c} , \\
X_{\Phi 2c} &= \left(\frac{r_b}{r}\right)^3 , \quad r \geqslant r_b , \\
X_{\Phi 2c} &= 2 - \left(\frac{r}{r_b}\right)^3 , \quad r \leqslant r_b ,
\end{aligned} \tag{11}$$

where $r_b = 3.44 \; kpc$, $v_c = 240 \; km \; s^{-1}$, $0.1 < Q_T < 0.4$ (Minchev & Famaey 2010).

At first, the second formula of Eqs. (11) does not hold. The inner part of the galactic bar potential must be calculated from the Poisson equation or its equivalence: $\triangle\Phi = 4\pi G\varrho$, $\Phi = - G \int \varrho(\vec{r}\,')/|\vec{r} - \vec{r}\,'| \, d^3\vec{r}\,'$. As for the case $r \geqslant r_b$ in Eqs. (11), it leads to

$$\begin{aligned}
\Phi^{(2)}_{conv}(x,y,z) &= \frac{1}{2} Q_T \, v_c^2 \left(\frac{r_b}{\sqrt{R^2 + z^2}}\right)^3 \frac{1}{R^2} \times Y_{\Phi 2c} \\
Y_{\Phi 2c} &= \left(x^2 - y^2\right) \cos(2\Omega_b t) \\
&\quad + 2 \, x \, y \, \sin(2\Omega_b t) .
\end{aligned} \tag{12}$$

We remind that $r = \sqrt{R^2 + z^2}$, where R is the 2-dimensional radial coordinate in the galactic equatorial plane and z is the vertical coordinate normal to the equatorial plane. One can immediately see that physical Eq. (10) differs from the conventional potential.

3   MODELS OF THE GALACTIC BAR

As examples of mass distribution in the galactic bar we take the models presented by Stanek et al. (1997). For the models G1, G3, E2, P1, P2, P3 we introduce the following complete nonsymmetric ellipsoid coordinates (r is a dimensionless quantity, now – it differs from the quantity used in the previous section):

$$x = r \, x_0 \, \sin\theta \, \cos\varphi ,$$

$$y = r \, y_0 \, \sin\theta \, \sin\varphi \, ,$$
$$z = r \, z_0 \, \cos\theta \, , \tag{13}$$

with the corresponding volume element

$$dx \, dy \, dz = x_0 \, y_0 \, z_0 \, r^2 \, \sin\theta \, d\theta \, d\varphi \, dr \, . \tag{14}$$

The corresponding quantities Q_{11} and Q_{22}, and, also mass of the galactic bar M, can now be calculated in an analytical way.

(i) The model G1 with mass density

$$\rho_{G1}(x,y,z) = \rho_0 \exp\left(-\,r^2/2\right) ,$$
$$r = \sqrt{\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 + \left(\frac{z}{z_0}\right)^2} \tag{15}$$

yields

$$\begin{aligned}
M &= 2\sqrt{2}\,\pi^{3/2}\,\rho_0\,x_0\,y_0\,z_0 \, , \\
Q_{11} &= 2\sqrt{2}\,\pi^{3/2}\,\rho_0\,x_0\,y_0\,z_0 \,\left(2x_0^2 - y_0^2 - z_0^2\right) \\
&= M\,\left(2x_0^2 - y_0^2 - z_0^2\right) \, , \\
\frac{Q_{22}}{Q_{11}} &= \frac{2y_0^2 - x_0^2 - z_0^2}{2x_0^2 - y_0^2 - z_0^2} \, .
\end{aligned} \tag{16}$$

(ii) The model G3 defined by the mass density

$$\rho_{G3}(x,y,z) = \tilde{\rho}_0 \, r^{-1.8} \exp\left(-\,r^3\right) , \tag{17}$$

produces

$$\begin{aligned}
M &= 4\pi \times 0.7394 \times \tilde{\rho}_0\,x_0\,y_0\,z_0 \, , \\
Q_{11} &\doteq \frac{4\pi}{3} \times 0.3219 \times \tilde{\rho}_0\,x_0\,y_0\,z_0 \,\left(2x_0^2 - y_0^2 - z_0^2\right) \\
&\doteq 0.1451 \times M\,\left(2x_0^2 - y_0^2 - z_0^2\right) \, , \\
\frac{Q_{22}}{Q_{11}} &= \frac{2y_0^2 - x_0^2 - z_0^2}{2x_0^2 - y_0^2 - z_0^2} \, ,
\end{aligned} \tag{18}$$

where the number 0.3219 is an approximate value of the integral

$$\int_0^\infty r^{2.2} \exp\left(-\,r^3\right) dr \, .$$

(iii) The model E2 defined by the mass density

$$\rho_{E2}(x,y,z) = \rho_0 \exp\left(-\,r\right) \tag{19}$$

gives

$$\begin{aligned}
M &= 8\pi\,\rho_0\,x_0\,y_0\,z_0 \, , \\
Q_{11} &\doteq 32\pi\,\rho_0\,x_0\,y_0\,z_0\,\left(2x_0^2 - y_0^2 - z_0^2\right) \\
&= 4\,M\,\left(2x_0^2 - y_0^2 - z_0^2\right) \, , \\
Q_{22}/Q_{11} &= \frac{2y_0^2 - x_0^2 - z_0^2}{2x_0^2 - y_0^2 - z_0^2} \, .
\end{aligned} \tag{20}$$

(iv) The model P1 with mass density

$$\rho_{P1}(x,y,z) = \frac{\rho_0}{(1+r)^4} \tag{21}$$

yields

$$\begin{aligned}
M &= 4\pi\,\rho_0\,x_0\,y_0\,z_0 \int_0^{r_{bar}} \frac{r^2}{(1+r)^4}\,dr \\
&= \frac{4\pi}{3}\,\rho_0\,x_0\,y_0\,z_0\,\frac{r_{bar}^3}{(1+r_{bar})^3} \, , \\
Q_{11} &= \frac{4\pi}{3}\,\rho_0\,x_0\,y_0\,z_0\,\left(2x_0^2 - y_0^2 - z_0^2\right) \int_0^{r_{bar}} \frac{r^4}{(1+r)^4}\,dr \\
&= \frac{4\pi}{3}\,\rho_0\,x_0\,y_0\,z_0\,\left(2x_0^2 - y_0^2 - z_0^2\right)\,\times X_{P1} \\
&= \frac{1}{3}\,M\,\left(2x_0^2 - y_0^2 - z_0^2\right)\left[3r_{bar} + 22 + \frac{30}{r_{bar}}\right. \\
&\left. + \frac{12}{r_{bar}^2} - 12\left(1 + \frac{1}{r_{bar}}\right)^3 \ln(1+r_{bar})\right] \, , \\
X_{P1} &= \frac{3r_{bar}^4 + 22r_{bar}^3 + 30r_{bar}^2 + 12r_{bar}}{3\,(1+r_{bar})^3} - 4\ln(1+r_{bar}) \\
r_{bar} &\equiv r_b/x_0 \quad (as\ an\ example)\, . \tag{22}
\end{aligned}$$

We have taken the symbol r_{bar} as the upper limit of the integral in Eq. (22) (if one would like to use infinity as the upper limit of the integral, then the integral would diverge, $Q_{11} \to \infty$. If we take into account that the real length of the bar r_b is proportional to x_0, then we can take $r_{bar} = r_b/x_0$.

(v) The model P2 with mass density

$$\rho_{P2}(x,y,z) = \frac{\rho_0}{r\,(1+r)^3} \tag{23}$$

yields

$$\begin{aligned}
M &= 4\pi\,\rho_0\,x_0\,y_0\,z_0 \int_0^{r_{bar}} \frac{r}{(1+r)^3}\,dr \\
&= 2\pi\,\rho_0\,x_0\,y_0\,z_0\,\frac{r_{bar}^2}{(1+r_{bar})^2} \, , \\
Q_{11} &= \frac{4\pi}{3}\,\rho_0\,x_0\,y_0\,z_0\,\left(2x_0^2 - y_0^2 - z_0^2\right) \int_0^{r_{bar}} \frac{r^3}{(1+r)^3}\,dr \\
&= \frac{4\pi}{3}\,\rho_0\,x_0\,y_0\,z_0\,\left(2x_0^2 - y_0^2 - z_0^2\right) \times X_{P2} \\
&= \frac{1}{3}\,M\,\left(2x_0^2 - y_0^2 - z_0^2\right)\left[2r_{bar} + 9 + \frac{6}{r_{bar}}\right. \\
&\left. - 6\left(1 + \frac{1}{r_{bar}}\right)^2 \ln(1+r_{bar})\right] \, , \\
X_{P2} &= \frac{2r_{bar}^3 + 9r_{bar}^2 + 6r_{bar}}{2\,(1+r_{bar})^2} - 3\ln(1+r_{bar}) \\
r_{bar} &\equiv r_b/x_0 \quad (as\ an\ example)\, . \tag{24}
\end{aligned}$$

(vi) The model P3 with mass density

$$\rho_{P3}(x,y,z) = \frac{\rho_0}{(1+r^2)^2} \tag{25}$$

yields

$$
\begin{aligned}
M &= 4\pi \ \rho_0 \ x_0 \ y_0 \ z_0 \int\limits_0^{r_{bar}} \frac{r^2}{(1+r^2)^2} \, dr \\
&= 2\pi \ \rho_0 \ x_0 \ y_0 \ z_0 \ \left(\arctan r_{bar} - \frac{r_{bar}}{1+r_{bar}^2} \right) , \\
Q_{11} &= \frac{4\pi}{3} \ \rho_0 \ x_0 y_0 z_0 \left(2x_0^2 - y_0^2 - z_0^2\right) \int\limits_0^{r_{bar}} \frac{r^4}{(1+r^2)^2} \, dr \\
&= \frac{4\pi}{3} \ \rho_0 \ x_0 \ y_0 \ z_0 \left(2x_0^2 - y_0^2 - z_0^2\right) \ \times \\
&\quad \left(r_{bar} + \frac{1}{2} \frac{r_{bar}}{1+r_{bar}^2} - \frac{3}{2} \arctan r_{bar} \right) \\
&= \frac{2}{3} \ M \ \left(2x_0^2 - y_0^2 - z_0^2\right) \ \times \\
&\quad \frac{r_{bar} + r_{bar}/\left[2\left(1+r_{bar}^2\right)\right] - (3/2)\arctan r_{bar}}{\arctan r_{bar} - r_{bar}/\left(1+r_{bar}^2\right)} , \\
r_{bar} &\equiv \ r_b/x_0 \quad (as \ an \ example) .
\end{aligned}
\tag{26}
$$

As for the model E1 with mass density

$$
\rho_{E1}(x,y,z) = \rho_0 \exp\left(-r_e\right) ,
\tag{27}
$$

where

$$
r_e = \frac{|x|}{x_0} + \frac{|y|}{y_0} + \frac{|z|}{z_0} ,
\tag{28}
$$

the values of M, Q_{11} a Q_{22} can be easily analytically calculated (one may avoid Eqs. 13 – 14):

$$
\begin{aligned}
M &= 8 \ \rho_0 \ x_0 \ y_0 \ z_0 , \\
Q_{11} &= 16 \ \rho_0 x_0 y_0 z_0 \left(2x_0^2 - y_0^2 - z_0^2\right) \\
&= 2M \ \left(2x_0^2 - y_0^2 - z_0^2\right) , \\
Q_{22}/Q_{11} &= \frac{2y_0^2 - x_0^2 - z_0^2}{2x_0^2 - y_0^2 - z_0^2} .
\end{aligned}
\tag{29}
$$

The model G2 with mass density

$$
\rho_{G2}(x,y,z) = \rho_0 \exp\left(-r_s^2/2\right) ,
\tag{30}
$$

where

$$
r_s = \left\{ \left[\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{y_0}\right)^2 \right]^2 + \left(\frac{z}{z_0}\right)^4 \right\}^{1/4} ,
\tag{31}
$$

can be solved with the following substitutions:

$$
\begin{aligned}
x &= x_0 \ r_s \ \sin^{1/2}\vartheta \ \cos\varphi , \\
y &= y_0 \ r_s \ \sin^{1/2}\vartheta \ \sin\varphi , \\
z &= z_0 \ r_s \ \cos^{1/2}\vartheta ,
\end{aligned}
\tag{32}
$$

for the upper hemisphere, and,

$$
\begin{aligned}
x &= x_0 \ r_s \ \sin^{1/2}\vartheta \ \cos\varphi , \\
y &= y_0 \ r_s \ \sin^{1/2}\vartheta \ \sin\varphi , \\
z &= -z_0 \ r_s \ \cos^{1/2}\vartheta ,
\end{aligned}
\tag{33}
$$

for the lower hemisphere, and, for both hemispheres $r_s \in \langle 0, \infty)$, $\vartheta \in \langle 0, \pi/2 \rangle$, $\varphi \in \langle 0, 2\pi)$, and, the volume element is

$$
dx \ dy \ dz = \frac{1}{2} \ x_0 \ y_0 \ z_0 \ r_s^2 \ \cos^{-1/2}\theta \ d\theta \ d\varphi \ dr_s
\tag{34}
$$

for both hemispheres. The values of the quantities M, Q_{11} and Q_{22}/Q_{11} are:

$$
\begin{aligned}
M &= 2 \ \pi^{3/2} \ K\left(\frac{1}{2}\right) \ \rho_0 \ x_0 \ y_0 \ z_0 , \\
Q_{11} &= 6\pi^{3/2}\sqrt{2} \ \rho_0 \ x_0 \ y_0 \ z_0 \times X_{G2} \\
&= 3\sqrt{2} \left[K\left(\frac{1}{2}\right) \right]^{-1} M \times X_{G2} , \\
\frac{Q_{22}}{Q_{11}} &= \frac{2y_0^2 - x_0^2 - \sqrt{2/\pi} \left[\Gamma\left(3/4\right)\right]^2 z_0^2}{2x_0^2 - y_0^2 - \sqrt{2/\pi} \left[\Gamma\left(3/4\right)\right]^2 z_0^2} , \\
X_{G2} &= 2x_0^2 - y_0^2 - \sqrt{\frac{2}{\pi}} \left[\Gamma\left(\frac{3}{4}\right) \right]^2 z_0^2 ,
\end{aligned}
\tag{35}
$$

where $K(x)$ is the complete elliptic integral of the first kind.

Finally, the model E3

$$
\rho_{E3}(x,y,z) = \rho_0 \ K_0(r_s) ,
\tag{36}
$$

yields

$$
\begin{aligned}
M &= \sqrt{2} \ \pi^2 \ K\left(\frac{1}{2}\right) \ \rho_0 \ x_0 \ y_0 \ z_0 , \\
Q_{11} &= 18 \ \pi^2 \ \rho_0 \ x_0 \ y_0 \ z_0 \times X_{E3} \\
&= 9\sqrt{2} \left[K\left(\frac{1}{2}\right) \right]^{-1} M \times X_{E3} , \\
\frac{Q_{22}}{Q_{11}} &= \frac{2y_0^2 - x_0^2 - \sqrt{2/\pi} \left[\Gamma\left(3/4\right)\right]^2 z_0^2}{2x_0^2 - y_0^2 - \sqrt{2/\pi} \left[\Gamma\left(3/4\right)\right]^2 z_0^2} , \\
X_{E3} &= 2x_0^2 - y_0^2 - \sqrt{\frac{2}{\pi}} \left[\Gamma\left(\frac{3}{4}\right) \right]^2 z_0^2 .
\end{aligned}
\tag{37}
$$

3.1   Numerical results

Table 1 presents numerical results for the data given by Stanek et al. (1997 – Table 4). Since the integrations in the models P1, P2 and P3 lead to integrals which require finite limits in order to be convergent, we have considered the upper limit corresponding to the length of the bar r_b and the values of x_0 for individual models (see Eqs. 22, 24, 26). Taking into account the value $r_b = 3.44 \ kpc$, the required quantities are presented in Table 2. If we admit a relative error of r_b to be 20 %, then the relative errors of the quantities are also presented in Table 2.

4   CONVENTIONAL AND PHYSICAL APPROACHES

In order to compare the published results with our approach, we have to summarize important equations.

Table 1. Numerical values of the quantities discussed in the previous subsection. Data for x_0, y_0 and z_0 are taken from Stanek et al. (1997). The quantity M/ρ_0 contains the central density ρ_0.

Model	x_0 [pc]	y_0 [pc]	z_0 [pc]
G1	1478	754	611
G3	4537	2303	1725
E2$^{(1)}$	1000	500	400
E2$^{(2)}$	1000	505	388
P1	1565	783	582
P2	2653	1320	975
P3	1869	939	713
E1	1650	868	474
G2	1298	694	572
E3	1075	563	442

Model	Q_{22}/Q_{11} [−]	Q_{11}/M [pc^2]	M/ρ_0 [pc^3]
G1	−0.4146	3.4271×10^6	1.0724×10^{10}
G3	−0.3938	4.7727×10^6	–
E2$^{(1)}$	−0.4151	6.3600×10^6	5.0266×10^9
E2$^{(2)}$	−0.4017	6.3777×10^6	4.9245×10^9
P1	−0.3957	–	–
P2	−0.3957	–	–
P3	−0.3999	–	–
E1	−0.3224	8.9338×10^6	5.4309×10^9
G2	−0.4461	5.7115×10^6	1.0639×10^{10}
E3	−0.4294	1.2084×10^7	6.9228×10^9

Table 2. Numerical values of quantities for the models P1, P2 and P3. The values of Q_{11}/M and M/ρ_0 hold for $r_b = 3.44$ kpc. The relative errors $\varrho(Q_{11}/M)$ and $\varrho(M/\rho_0)$ hold for the uncertainty of 20 % in r_b.

Model	Q_{22}/Q_{11} [−]	Q_{11}/M [pc^2]	$\varrho(Q_{11}/M)$ [%]
P1	−0.3957	2.3149×10^6	−31 / +34
P2	−0.3957	2.1628×10^6	−32 / +36
P3	−0.3999	2.5152×10^6	−29 / +31

Model	Q_{22}/Q_{11} [−]	M/ρ_0 [pc^3]	$\varrho(M/\rho_0)$ [%]
P1	−0.3957	0.9700×10^9	−20 / +17
P2	−0.3957	6.8383×10^9	−19 / +16
P3	−0.3999	5.1389×10^9	−22 / +18

4.1 Conventional approach

The conventional approach to galactic motion of the Sun defines the following potentials of the Galaxy (Minchev et al. 2010). The background axisymmetric potential due to the disk and halo has the form

$$\Phi_{0\ conv} = v_c^2 \ \ln(r/r_0) \tag{38}$$

corresponding to a flat rotation curve. The non-axisymmetric potential due to the galactic bar is modeled as a pure quadrupole

$$\Phi_{b\ conv}^{(2)} = \frac{1}{2} \ Q_T \ v_c^2 \ \cos\left[2(\phi - \Omega_b t)\right] \times X_{\Phi_{b\ conv}^{(2)}} \ ,$$

$$X_{\Phi_{b\ conv}^{(2)}} = \left(\frac{r_b}{r}\right)^3 \ , \ \ r \geqslant r_b \ ,$$

$$X_{\Phi_{b\ conv}^{(2)}} = \left[2 - \left(\frac{r}{r_b}\right)^3\right] \ , \ \ r \leqslant r_b \ , \tag{39}$$

where $r_b = 3.44$ kpc, $v_c = 240$ km s^{-1}, $0.1 < Q_T < 0.4$ (see also Minchev & Famaey 2010) and the spiral potential is given by

$$\Phi_{s\ conv} = \epsilon_s \ \cos\left[\tilde{\alpha} \ \ln\left(r/r_0\right) \ - \ m(\phi - \Omega_s t)\right] \ , \tag{40}$$

where ϵ_s is the spiral structure strength, $0.015 < |\epsilon_s| < 0.072$ for a two-armed spiral structure and $0.007 < |\epsilon_s| < 0.031$ for a four-armed spiral structure, $\epsilon_s = sign(-\ \tilde{\alpha}) \times |\epsilon_s|$, $\tilde{\alpha} = -4$ and -8 for the two-armed and four-armed spiral structure, respectively, where the negative sign corresponds to trailing spirals, m is an integer corresponding to the number of arms, $\Omega_s \in \{0.7, 0.8, 0.9, 1.0, 1.1\} \times v_c/R_0$, i.e., $\Omega_s[km \ s^{-1} \ kpc^{-1}] \in \{21, 24, 27, 30, 33\}$, if $v_c = 240$ km s^{-1} and $R_0 = 8.0$ kpc.

4.2 Physical approach

In this section we take the background axisymmetric potential due to the disk and halo in the form similar to Eq. (38)

$$\Phi_{disk+halo} = -\ v_c^2 \ \{1 \ - \ \ln(r/r_G)\} \ , \tag{41}$$

where r_G is the dark matter radius of the Galaxy. Eq. (41) is the physical potential corresponding to the flat rotation curve defined by the condition $v(r) = v_c$. The result presented by Minchev & Famaey (2010) or Dehnen (2000), see our Eq. (38), differs from Eq. (41).

The potential due to the galactic bar is given by Eqs. (3) and (10):

$$\Phi_{bar}(\vec{r}) = \Phi^{(0)}(\vec{r}) \ + \ \Phi^{(2)}(\vec{r}) \ ,$$

$$\Phi^{(0)}(\vec{r}) = -\ G \ \frac{1}{r} \int \varrho(\vec{r}\,') \ d^3\vec{r}\,' = -\ \frac{G \ M}{r} \ ,$$

$$\Phi^{(2)}(x,y,z) = -\ \frac{G \ Q_{11} \ (1 - Q_{22}/Q_{11})}{4 \ r^5}$$
$$\times \left[\frac{1 + Q_{22}/Q_{11}}{1 - Q_{22}/Q_{11}} \left(x^2 + y^2 - 2\ z^2\right) \right.$$
$$\left. + \ \left(x^2 - y^2\right) \cos\left(2\alpha\right) + 2xy \sin\left(2\alpha\right)\right] \ ,$$

$$M = 1.4 \times 10^{10} M_\odot \ ,$$
$$Q_{11}[M_\odot \ pc^2] = 8 \times 10^6 \ \times M[M_\odot] \ ,$$
$$Q_{22}/Q_{11} = -\ 2/5 \ ,$$
$$\alpha = \Omega_b \ t \ + \ \alpha_0 \ ,$$
$$\alpha_0 \doteq -\ 25° \ ,$$
$$\Omega_b \doteq -\ 55.5 \ km \ s^{-1} \ kpc^{-1} \ , \tag{42}$$

where also the value of M from Dauphole & Colin (1995 - Table 5) is used and the values from Table 1 are taken into account.

The spiral potential is given by the conventional model represented by Eq. (40), in a slightly corrected form ($R_b =$ 3.44 kpc):

$$
\begin{aligned}
\Phi_{s\ corr} &= \epsilon_s v_c^2 \cos\left[\tilde{\alpha}\ \ln\left(\ R/R_b\right)\ -\ m(\phi - \phi_{s0} + \Omega_s t)\right] \\
&= \epsilon_s v_c^2 \left\{ S_1 \cos\left(m\phi_{s0} + \tilde{\alpha}\ln\frac{R}{R_b}\right)\right. \\
&\quad \left. +\ S_2 \sin\left(m\phi_{s0} + \tilde{\alpha}\ln\frac{R}{R_b}\right)\right\}\ , \\
S_1 &= \cos\left(m\phi\right)\cos\left(m\Omega_s t\right) - \sin\left(m\phi\right)\sin\left(m\Omega_s t\right)\ , \\
S_2 &= \sin\left(m\phi\right)\cos\left(m\Omega_s t\right) + \cos\left(m\phi\right)\sin\left(m\Omega_s t\right)\ , \\
\cos\left(4\phi\right) &= 4\left[\left(\frac{x}{R}\right)^4 +\ \left(\frac{y}{R}\right)^4\right] - 3\ , \\
\sin\left(4\phi\right) &= 4\ \frac{x}{R}\ \frac{y}{R}\ \left[\left(\frac{x}{R}\right)^2 -\ \left(\frac{y}{R}\right)^2\right]\ , \\
\cos\left(2\phi\right) &= \left(\frac{x}{R}\right)^2 -\ \left(\frac{y}{R}\right)^2\ , \\
\sin\left(2\phi\right) &= 2\ \frac{x}{R}\ \frac{y}{R}\ , \\
R &= \sqrt{x^2 + y^2}\ , \\
\phi_{s0} &= \alpha_0\ , \tag{43}
\end{aligned}
$$

where the quantity ϕ_{s0} denotes orientation of spiral arms in the coordinates x-y at the intial time $t = 0$ and we put $\phi_{s0} = \alpha_0$. The quantity ϵ_s is the spiral structure strength, $0.015 < |\epsilon_s| < 0.072$ for a two-armed spiral structure and $0.007 < |\epsilon_s| < 0.031$ for a four-armed spiral structure, $\epsilon_s = sign(\tilde{\alpha}) \times |\epsilon_s|$, $\tilde{\alpha} = +4$ and $+8$ for the two-armed and four-armed spiral structure, respectively, where the positive sign corresponds to trailing spirals, m is an integer corresponding to the number of arms, $\Omega_s \in \{0.7, 0.8, 0.9, 1.0, 1.1\} \times v_{y0}/R_0$, where v_{y0} is the initial circular velocity of the Sun and $R_0 = 8.0\ kpc$.

4.3 Equation of motion

If an object moves in the Galaxy under the action of galactic disk, halo, galactic bar and spiral structure, then the equation of motion of the object reads

$$
\dot{\vec{v}} = - grad\left(\Phi_{disk+halo} + \Phi_{bar} + \Phi_{s\ corr}\right)\ , \tag{44}
$$

where Eqs. (41), (42) and (43) can be used.

4.4 Motion of the Sun – initial conditions

Initial conditions for the Sun are

$$
\begin{aligned}
x &= R_0 = 8\ kpc\ , \\
y &= 0\ , \\
v_x &= 0\ , \\
v_y &= v_{y0} = -255.2\ km\ s^{-1}\ ,
\end{aligned}
$$

Table 3. Numerical values of minimum and maximum galactocentric distances of the Sun under the action of galactic bar, disk and halo. Motion for the time span of 5×10^9 years is considered.

v_c [$km\ s^{-1}$]	v_{y0} [$km\ s^{-1}$]	R_{min} [kpc]	R_{max} [kpc]
240.000	−255.196	7.384	9.193
202.176	−220.000	7.774	8.317

$$
\begin{aligned}
&\quad for\ Eqs.\ (41,\ 42,\ 43)\ -\ the\ first\ case\ , \\
v_y &= v_{y0} = -\ 220.0\ km\ s^{-1}\ , \\
&\quad for\ Eqs.\ (41,\ 42,\ 43)\ -\ the\ second\ case\ , \\
v_y &= v_{y0} = +\ 240.0\ km\ s^{-1}\ , \\
&\quad for\ Eqs.\ (38,\ 39,\ 40)\ , \tag{45}
\end{aligned}
$$

where the value of v_y follows from the relations

$$
\begin{aligned}
v_y &= -\sqrt{v_c^2 + \Sigma(v^2)}\ , \\
\Sigma(v^2) &= \left([v(R_0)]^2\right)_{\Phi^{(0)}} + \left([v(R_0)]^2\right)_{\Phi^{(2)}}\ , \\
\left([v(R_0)]^2\right)_{\Phi^{(0)}} &= \left[R\ \frac{\partial\Phi^{(0)}}{\partial R}\right]_{R_0} = \frac{GM}{R_0}\ , \\
\left([v(R_0)]^2\right)_{\Phi^{(2)}} &= \left[R\ \frac{\partial}{\partial R}\left\{-\ \frac{G\ Q_{11}}{4R^3}\left(1 + \frac{Q_{22}}{Q_{11}}\right)\right\}\right]_{R_0} \\
&= \frac{3}{32}\left(1 + \frac{Q_{22}}{Q_{11}}\right) \times \frac{GM}{R_0}\ , \tag{46}
\end{aligned}
$$

for the physical approach. Eqs. (38)-(45) uniquely determine motion of the Sun in the Galaxy.

5 COMPUTATIONAL RESULTS

We numerically solved equation of motion in accordance with Eq. (44) and the results presented in the previous section. We used both of the physical values of v_y, as it is given by Eqs. (45 – the first and the second cases).

5.1 Effect of the galactic bar

Table 3 shows the extremal values of the evolution of the galactocentric distance of the Sun under the action of galactic halo, disk and galactic bar. The results presented in Table 3 hold for the special case when the effect of spiral structure is ignored. While the nonexistence of the nonsymmetric parts in term $\Phi^{(2)}$ would produce circular orbit for the initial conditions given by Eqs. (45 - the first and the second cases), $R = R_0 = 8\ kpc$, the nonsymmetric parts in the term $\Phi^{(2)}$ cause slight oscillations in $R(t)$.

The relative change in R is about 15 % (or even larger) according to Famaey & Minchev (2010), Minchev & Famaey (2010). Our physical approach yields two different values. The first case, when $v_{y0} = -255.196\ km\ s^{-1}$, the initial orbital velocity of the Sun, yields $R_{min}/R_0 = 0.923$ and

Table 4. Numerical values of minimum and maximum galactocentric distances of the Sun under the action of galactic bar, disk and halo. Motion for the time span of 5×10^9 years is considered, $v_{y0} = -220.0 \; km \; s^{-1}$.

Q_{11}/M [pc^2]	v_c [$km \; s^{-1}$]	R_{min} [kpc]	R_{max} [kpc]
3.4271×10^6	202.742	7.752	8.300
1.2048×10^7	201.669	7.796	8.333

$R_{max}/R_0 = 1.149$. While the value of R_{max} corresponds to the value given by Famaey & Minchev (2010), the value of R_{min} is greater than the value presented by other authors. Moreover, we have to stress that the effect of spiral arms considered by Minchev & Famaey (2010), Famaey & Minchev (2010), Minchev et al. (2010), Minchev et al. (2011) is not considered in our calculations. The second case, when $v_{y0} = -220.0 \; km \; s^{-1}$, yields $R_{min}/R_0 = 0.972$ and $R_{max}/R_0 = 1.040$. The galactocentric distance of the Sun changes in much smaller interval than it was presented in the previous papers. Again, the effect of spiral arms is not considered in our calculations.

Let us consider motion of the Sun during 5×10^9 years. The first case, characterized by the value $v_c = 240 \; km \; s^{-1}$, yields that Sun moves within galactocentric distance $R \in \langle 7.75 \; kpc, 8.25 \; kpc \rangle$ during 44.3% of the total integration time, and, $R \in \langle 7.5 \; kpc, 8.5 \; kpc \rangle$ during 71.5% of the total time. The second case, characterized by $v_{y0} = -220.0 \; km \; s^{-1}$, yields that Sun moves within galactocentric distance $R \in \langle 7.9 \; kpc, 8.1 \; kpc \rangle$ during 50.5% of the total time, and, $R \in \langle 7.8 \; kpc, 8.2 \; kpc \rangle$ during 83.2% of the total time. The results do not change if we consider the total time span 10×10^9 years instead of 5×10^9 years.

In reality, there exists some dispersion in the values of Q_{11}. Taking into account the results of Table 1, we can consider the following two extremal values: $Q_{11}/M = 3.4271 \times 10^6 \; pc^2$ for the model G1, and, $Q_{11}/M = 1.2048 \times 10^7 \; pc^2$ for the model E3. The results for the radial migration of the Sun are summarized in Table 4. The Table 4 shows that the values are: $R_{min} \geqslant 7.75 \; kpc$ and $R_{max} \leqslant 8.33 \; kpc$.

5.2 Simultaneous effect of the galactic bar and the spiral structure

If we take into account Eqs. (43), then galactic radial migration of the Sun is less relevant than Minchev & Famaey (2010), Famaey & Minchev (2010), Minchev et al. (2010), Minchev et al. (2011) present. This holds both for the two- and four-armed spiral structures, although the effect of the two-armed spiral structure is much more relevant than the effect of the four-armed spiral structure, see Tables 5 and 7.

The results summarized in Tables 6 and 7 show that radial migration of the Sun is practically unaffected by the effect of the four-armed spiral structure with $v_c = 202.176 \; km \; s^{-1}$. The results are practically independent on the spiral

Table 5. Numerical values of minimum and maximum galactocentric distances of the Sun under the action of galactic bar, disk + halo and spiral arms. The two-armed spiral structure and $v_c = 202.176 \; km \; s^{-1}$, $v_{y0} = -220.0 \; km \; s^{-1}$ and $v_{x0} = 0.0 \; km \; s^{-1}$ are considered. The results hold for the time integration of 5×10^9 years. The Sun remains $(T \; / \; 100) \times 5 \times 10^9$ years in the galactocentric distances $R \in \langle 7.8, \; 8.2 \rangle \; kpc$.

ϵ_s [−]	Ω_s [v_{y0}/R_0]	R_{min} [kpc]	R_{max} [kpc]	T [%]
0.015	0.7	7.693	8.266	84.9
	0.8	7.759	8.225	93.3
	0.9	7.749	8.223	93.1
	1.0	7.758	8.232	94.5
	1.1	7.744	8.224	92.8
0.030	0.7	7.434	8.445	52.7
	0.8	7.632	8.248	76.5
	0.9	7.673	8.197	82.6
	1.0	7.658	8.141	74.9
	1.1	7.696	8.164	85.4
0.045	0.7	7.109	8.752	39.0
	0.8	7.434	8.362	51.1
	0.9	7.519	8.298	60.4
	1.0	7.509	8.225	59.3
	1.1	7.582	8.200	67.5
0.060	0.7	6.772	8.971	29.3
	0.8	7.201	8.534	37.0
	0.9	7.339	8.459	44.2
	1.0	7.333	8.346	44.3
	1.1	7.437	8.268	50.0

structure strength. The found interval for radial migration of the Sun $(7.8 − 8.4) \; kpc$ is much shorter than the result presented by Minchev & Famaey (2010), Famaey & Minchev (2010), Minchev et al. (2010), Minchev et al. (2011). We can explain the difference by incorrectness of the equations used by the above mentioned authors. Moreover, it seems to be nonphysical to use the case of resonance $\Omega_s = v_{y0}/R_0$ as the relevant for the motion of the Sun in the Galaxy. There is no argument why the spiral arms should rotate with the angular velocity equal to the angular velocity of the Sun. The well-known result states that a flat rotation curve exists for the Galaxy. The flat rotation curve yields an orbital speed practically independent on the galactocentric distance and the angular velocity is given by the ratio of the orbital speed and the galactocentric distance. Thus, the angular speed of a galactic object is a function of the galactocentric distance and the fixed value of Ω_s, $\Omega_s = v_{y0}/R_0$, is a pure mathematical toy and it contains no relevant physics.

The last column in Tables 5-7 denotes the relative time T of the existence of the Sun in the galactocentric distances $R \in \langle 7.8, \; 8.2 \rangle \; kpc$. The two-armed spiral structure yields $T < 50\%$ for large values of spiral structure strength. The four-armed spiral structure yields $T \in (69\%, 79\%)$.

Table 6. Numerical values of minimum and maximum galactocentric distances of the Sun under the action of galactic bar, disk + halo and spiral arms. The four-armed spiral structure, $v_c = 202.176\ km\ s^{-1}$, $v_{y0} = -220.0\ km\ s^{-1}$ and $v_{x0} = 0.0\ km\ s^{-1}$ are considered. The results hold for the time integration of 1×10^9 years. The Sun remains $(T\ /\ 100) \times 1 \times 10^9$ years in the galactocentric distances $R \in \langle 7.8,\ 8.2 \rangle\ kpc$.

ϵ_s [−]	Ω_s [v_{y0}/R_0]	R_{min} [kpc]	R_{max} [kpc]	T [%]
	0.7	7.769	8.297	77.6
	0.8	7.774	8.311	78.6
0.007	0.9	7.769	8.308	78.5
	1.0	7.779	8.300	78.2
	1.1	7.772	8.310	78.8
	0.7	7.764	8.299	72.4
	0.8	7.769	8.328	75.4
0.014	0.9	7.766	8.322	74.7
	1.0	7.783	8.309	74.8
	1.1	7.770	8.326	75.8
	0.7	7.758	8.329	69.9
	0.8	7.765	8.345	73.9
0.021	0.9	7.763	8.334	74.2
	1.0	7.781	8.323	73.2
	1.1	7.768	8.339	74.3
	0.7	7.752	8.360	69.1
	0.8	7.759	8.381	72.7
0.028	0.9	7.761	8.357	72.4
	1.0	7.778	8.340	72.9
	1.1	7.763	8.355	73.3

Table 7. Numerical values of minimum and maximum galactocentric distances of the Sun under the action of galactic bar, disk + halo and spiral arms. The four-armed spiral structure, $v_c = 202.176\ km\ s^{-1}$, $v_{y0} = -220.0\ km\ s^{-1}$ and $v_{x0} = 0.0\ km\ s^{-1}$ are considered. The results hold for the time integration of 5×10^9 years. The Sun remains $(T\ /\ 100) \times 5 \times 10^9$ years in the galactocentric distances $R \in \langle 7.8,\ 8.2 \rangle\ kpc$.

ϵ_s [−]	Ω_s [v_{y0}/R_0]	R_{min} [kpc]	R_{max} [kpc]	T [%]
	0.7	7.769	8.343	79.9
	0.8	7.769	8.339	80.2
0.007	0.9	7.770	8.338	79.9
	1.0	7.776	8.340	79.8
	1.1	7.772	8.333	80.2
	0.7	7.763	8.372	78.6
	0.8	7.769	8.366	79.6
0.014	0.9	7.766	8.361	79.1
	1.0	7.778	8.362	78.3
	1.1	7.767	8.359	78.9
	0.7	7.758	8.394	77.7
	0.8	7.763	8.396	77.9
0.021	0.9	7.764	8.389	77.9
	1.0	7.778	8.385	77.2
	1.1	7.765	8.376	78.5
	0.7	7.752	8.422	75.7
	0.8	7.756	8.419	75.6
0.028	0.9	7.759	8.415	76.1
	1.0	7.771	8.406	76.2
	1.1	7.762	8.395	77.2

6   DISCUSSION

The first case treated in the previous section, $v_c = 240\ km\ s^{-1}$, corresponds to the value used by, e.g., Minchev & Famaey (2010), Famaey & Minchev (2010), Minchev et al. (2010), Minchev et al. (2011). The second approach characterized by $v_{y0} = -220.0\ km\ s^{-1}$ is the conventional approach, it uses the conventional speed (e.g., recommended by IAU) of the Sun $220.0\ km\ s^{-1}$. As a consequence, the radial galactic migration of the Sun is smaller than ± 0.32 kpc during the interval $(5-10) \times 10^9$ years. Thus, the radial migration found by Minchev & Famaey (2010), Famaey & Minchev (2010), Minchev et al. (2010), Minchev et al. (2011) is caused not only by an incorrect equation of motion, but also by the fact that the effect of galactic bar together with other galactic components is sensitive to the real velocity of the Sun.

Which of the two values of v_{y0} is more realistic? We can answer this question by calculating the Oort constants. The first case, corresponding to $v_c = 240\ km\ s^{-1}$, yields $A = 17.1$ $km\ s^{-1}\ kpc^{-1}$ and $B = -14.9\ km\ s^{-1}\ kpc^{-1}$, if the galactic disk, halo (Eq. 41) and the dominant parts of the galactic bar (Eq. 42 – $\Phi^{(0)}(\vec{r})$, $\Phi^{(2)}(\vec{r})$) are considered. The second case, $v_{y0} = -220.0\ km\ s^{-1}$ yields $A = 15.0\ km\ s^{-1}\ kpc^{-1}$

Table 8. Numerical values of minimum and maximum galactocentric distances of the Sun under the action of galactic bar, disk + halo and spiral arms. The four-armed spiral structure and $v_c = 202.176\ km\ s^{-1}$ are considered. The results hold for the time integration of 5×10^9 years. The initial $x-$ component of the Sun's velocity is v_x. The negative sign of v_x corresponds to the orientation toward the center of the Galaxy. The case when the resonance between the rotation of the Sun and spiral arms is considered: $\epsilon_s = 0.028$, $\Omega_s = 1.0\ v_{y0}/R_0$, $v_{y0} = 220.0\ km\ s^{-1}$ and $R_0 = 8\ kpc$.

v_x [$km\ s^{-1}$]	R_{min} [kpc]	R_{max} [kpc]
+ 10.0	7.558	8.752
0.0	7.771	8.405
− 10.0	7.774	8.429

and $B = -12.5\ km\ s^{-1}\ kpc^{-1}$. The first case is not consistent with the most probably values of the Oort constants, e.g., Olling & Merrifield (1998) yield $A = (11\text{-}15)\ km\ s^{-1}\ kpc^{-1}$, $B = $ -(12-14) $km\ s^{-1}\ kpc^{-1}$ (see also Lequeux 2005, p. 10). Thus, the physical result should correspond to $v_{y0} = -220.0$ $km\ s^{-1}$. Galactic migration of the Sun is less than ± 0.4

Table 9. Numerical values of minimum and maximum galacto-centric distances of the Sun under the action of galactic bar, disk + halo and spiral arms. The two-armed spiral structure and v_c = 202.176 $km\ s^{-1}$ are considered. The results hold for the time integration of 5×10^9 years. The initial $x-$ component of the Sun's velocity is v_x. The negative sign of v_x corresponds to the orientation toward the center of the Galaxy. The case when the resonance between the rotation of the Sun and spiral arms is considered: $\epsilon_s = 0.06$, $\Omega_s = 1.0\ v_{y0}/R_0$, $v_{y0} = 220.0\ km\ s^{-1}$ and R_0 = 8 kpc.

v_x $[km\ s^{-1}]$	R_{min} $[kpc]$	R_{max} $[kpc]$
+ 10.0	7.324	8.436
0.0	7.333	8.346
− 10.0	7.120	8.667

kpc with respect to the current value 8.0 kpc for the four-armed spiral structure. If we would add a nonzero value of radial velocity component to the velocity of the Sun, then the radial migration might be more relevant, see Table 8. However, the two-armed spiral structure yields much greater radial migration, up to 1.2 kpc for the largest value of the spiral structure strength. If we would add a nonzero value of radial velocity component to the velocity of the Sun, then the radial migration might be more relevant, see Table 9.

7   CONCLUSION

The paper presents detailed derivation of the relevant equations of motion for the Sun under the action of galactic bar and spiral arms. The equations differ from the equations used by other authors.

Improving physics presented by Dehnen (2000), Minchev & Famaey (2010), Famaey & Minchev (2010), Minchev et al. (2010), Minchev et al. (2011) and others, we have obtained significantly different radial migration of the Sun than the previous authors.

Galactic migration of the Sun is less than $\pm$ 0.4 kpc, if the four-armed spiral structure and the value $v_{y0} = -220.0$ $km\ s^{-1}$ are considered. The Sun remains $(69 - 79)\%$ of its existence within the galactocentric distances $(7.8 - 8.2)$ kpc and the result is practically insensitive on the spiral structure strength. Thus, the radial distance changes only within 5% from the value of 8 kpc.

Galactic radial migration of the Sun is less than $\pm$ $(0.3 - 1.2)$ kpc for the two-armed spiral structure and $v_{y0} = -220.0$ $km\ s^{-1}$. The Sun remains $(29 - 95)\%$ of its existence within galactocentric distances $(7.8 - 8.2)$ kpc and the results strongly depend on the spiral structure strength and the angular speed of the spiral arms. The radial distance changes within $(3.8 - 15.0)\%$ from the value of 8 kpc.

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